

Skill Enhancement Courses (SECs) for Semester -V,

From 2022-23(Syllabus-Curriculum) <u>Structure of SECs for Semester-V</u>

(To choose One pair from the Four alternate pairs of SECs)

Univ	Course	Name of Course	Hours/	Credits	Marks	
	Number		Week			
Code	6&7				IA-20	Sem
					Filed Work 05	End
	6A	Numerical Methods	6	5	25	75
	7A	Mathematical Special Functions	6	5	25	75

OR

6B	Multiple integrals and Applications of Vector Calculus	6	5	25	75
7B	Integral transforms with Applications	6	5	25	75

OR

6C	Partial Differential Equations and Fourier Series	6	5	25	75
7C	Number theory	6	5	25	75

Note: *Course type code: T: Theory, L: Lab, P: Problem solving

*Note: FIRST and SECOND PHASES (2 spells) of APPRENTICESHIP between 1st and 2nd year and between 2nd and 3rd year (two summer vacations)

*Note: THIRD PHASE of APPRENTICESHIP Entire 5th / 6th Semester

Note-1: For Semester–V, for the domain subject Mathematics, any one of the three pairs of SECs shall be chosen as courses 6 and 7, i.e., (6A & 7A) or (6B & 7B) or (6C & 7C), the pair shall not be broken. A, B, C allotment is random, not on any priority basis.

Note-2: One of the main objectives of Skill Enhancement Courses (SEC) is to inculcate skills related to the domain subject in students. The syllabus of SEC will be partially skill oriented. Hence, teachers shall also impart practical training to students on the skills embedded in the syllabus citing related real field situations.



B. A/B.Sc	Semester – V (Skill Enhancement Course- Elective)	Credits:
Course: 6A	Numerical Methods	Hrs/Wk:

Learning Outcomes:

Students after successful completion of the course will be able to

- 1. understand the subject of various numerical methods that are used to obtain approximate solutions
- 2. Understand various finite difference concepts and interpolation methods.
- 3. Work out numerical differentiation and integration whenever and wherever routine methods are not applicable.
- 4. Find numerical solutions of ordinary differential equations by using various numerical methods.
- 5. Analyze and evaluate the accuracy of numerical methods.

Syllabus : (Hours: Teaching: 75 (incl. unit tests etc. 05), Training: 15)

Unit – 1: Finite Differences and Interpolation with Equal intervals (15h)

- 1. Introduction, Forward differences, Backward differences, Central Differences, Symbolic relations, nth Differences of Some functions,
- 2. Advancing Difference formula, Differences of Factorial Polynomial.
- 3. Newton's formulae for interpolation. Central Difference Interpolation Formulae.

Unit – 2: Interpolation with Equal and Unequal intervals

1. Central Difference Interpolation Formulae.

Gauss's Forward interpolation formula, Gauss's backward interpolation formula, Stirling's formula, Bessel's formula.

(15h)

- 2. Interpolation with unevenly spaced points, divided differences and properties, Newton's divided differences formula.
- 3. Lagrange's interpolation formula, Lagrange's Inverse interpolation formula.

Unit – 3: Numerical Differentiation (15h)

- 1. Derivatives using Newton's forward difference formula, Newton's back ward difference formula,
- 2. Derivatives using central difference formula, Stirling's interpolation formula,
- 3. Newton's divided difference formula, Maximum and minimum values of a tabulated function.

Unit – 4: Numerical Integration (15h)

- 1. General quadrature formula one errors, Trapezoidal rule,
- 2. Simpson's 1/3–rule, Simpson's 3/8 rule, and Weddle's rules,
- 3. Euler McLaurin Formula of summation and quadrature, The Euler transformation.

Unit – 5: Numerical solution of ordinary differential equations (15h)

- 1. Introduction, Solution by Taylor's Series,
- 2. Picard's method of successive approximations,
- **3.** Euler's method, Modified Euler's method, Runge Kutta methods.

References:

- 1. S.S.Sastry, Introductory Methods of Numerical Analysis, Prentice Hall of India Pvt. Ltd., New Delhi-110001, 2006.
- 2. P.Kandasamy, K.Thilagavathy, Calculus of Finite Differences and Numerical Analysis.
 - S. Chand & Company, Pvt. Ltd., Ram Nagar, New Delhi-110055.
- 3. R.Gupta, Numerical Analysis, Laxmi Publications (P) Ltd., New Delhi.
- 4. H.C Saxena, Finite Differences and Numerical Analysis, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
- 5. S.Ranganatham, Dr.M.V.S.S.N.Prasad, Dr.V.Ramesh Babu, Numerical Analysis,
 - S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
- 6. Web resources suggested by the teacher and college librarian including reading material.

Co-Curricular Activities:

A) Mandatory:

- **1. For Teacher:** Teacher shall train students in the following skills for 15 hours, by taking relevant outside data (Field/Web).
- 1. Applications of Newton's forward and back ward difference formulae.
- 2. Applications of Gauss forward and Gauss back ward, Stirling's and Bessel's formulae.
- 3. Applications of Newton's divided differences formula and Lagrange's interpolation formula.
- 4. Various methods to find the approximation of a definite integral.
- 5. Different methods to find solutions of Ordinary Differential Equations.
- **2.** For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work and submit a report not exceeding 10 pages in the given format on the workdone in the areas like the following, by choosing any one of the aspects.
 - 1. Collecting the data from the identified sources like Census department or Electricity department, by applying the Newton's, Gauss and Lagrange's interpolation formula, making observations and drawing conclusions. (Or)
 - 2. Selection of some region to find the area by applying Trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule, and Weddle's rules. Comparing the solutions with analytical solution and concluding which one is the best method. (Or)
- 3. Finding solution of the ODE by Taylor's Series, Picard's method of successive approximations, Euler's method, Modified Euler's method, Runge-Kutta methods. Comparing the solutions with analytical solution, selecting the best method.
- 3. Max. Marks for Fieldwork/Project work Report: 05.
- **4.** Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.
- 5. Unit tests (IE).

b) Suggested Co-Curricular Activities:

- 1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
- 2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
- 3. Invited lectures and presentations on related topics by experts in the specified area.



B. A/B.Sc	Semester – V (Skill Enhancement Course- Elective)	Credits:
Course: 7A	Mathematical Special Functions	Hrs/Wk:

Learning Outcomes:

Students after successful completion of the course will be able to:

- 1. Understand the Beta and Gamma functions, their properties and relation between these two functions, understand the orthogonal properties of Chebyshev polynomials and recurrence relations.
- 2. Find power series solutions of ordinary differential equations.
- 3. solve Hermite equation and write the Hermite Polynomial of order (degree) n, also find the generating function for Hermite Polynomials, study the orthogonal properties of Hermite Polynomials and recurrence relations.
- 4. Solve Legendre equation and write the Legendre equation of first kind, also find the generating function for Legendre Polynomials, understand the orthogonal properties of Legendre Polynomials.
- 5. Solve Bessel equation and write the Bessel equation of first kind of order n, also find the generating function for Bessel function understand the orthogonal properties of Bessel function.

Syllabus: (Hours: Teaching: 75 (incl. unit tests etc. 05), Training: 15)

Unit – 1: Beta and Gamma functions.

(15h)

- 1. Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions.
- 2. Transformation of Gamma Functions. Another form of Beta Function,
- 3. Relation between Beta and Gamma Functions.

CHAPTER: 2.9 to 2.15 of Prescribed text book 1

Unit – 2: Power series and Power series solutions of ordinary differential equations (15h)

- 1. Introduction, summary of useful results, power series, radius of convergence, theorems on Power series
- 2. Introduction of power series solutions of ordinary differential equation
- 3. Ordinary and singular points, regular and irregular singular points, power series solution.

CHAPTER: 7.1to 7.8 and 8.1 to 8.6 of Part-II of Prescribed text book 2

Unit – 3: Hermite polynomials

(15h)

- 1. Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials, generating function for Hermite polynomials.
- 2. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few Hermite Polynomials.
- 3. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

CHAPTER: 6.1 to 6.8 of Prescribed text book 1

Unit – 4: Legendre polynomials

(15h)

- 1. Definition, Solution of Legendre's equation, Legendre polynomial of degree n, generating function of Legendre polynomials.
- 2. Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation (derivations not required) to show that $P_n(x)$ is the coefficient of h^n , in the expansion of $(1 2xh + h^2)^{\frac{-1}{2}}$
- 3. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

CHAPTER: 2.1 to 2.8 and 2.12 of Prescribed text book 1

Unit – 5: Bessel's equation

(15h)

- 1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n, Bessel's function of the second kind of order n.
- 2. Integration of Bessel's equation in series form=0, Definition of $J_n(x)$, recurrence formulae for $J_n(x)$.
- 3. Generating function for $J_n(x)$.

CHAPTER: 5.1 to 5.7 of Prescribed text book 1

Prescribed Books:

- 1. Dr.M.D.Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
- 2. J.N.Sharma and Dr.R.K.Gupta, Differential equations with special functions, Krishna Prakashan Mandir.

Reference Books:

- 1. Shanti Narayan and Dr.P.K.Mittal, Integral Calculus, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
- 2. George F.Simmons, Differential Equations with Applications and Historical Notes, Tata McGRAW-Hill Edition, 1994.
- 3. Shepley L.Ross, Differential equations, Second Edition, John Willy & sons, New York, 1974.
- 4. Web resources suggested by the teacher and college librarian including reading material.

Co-Curricular Activities:

A) Mandatory:

- **1. For Teacher:** Teacher shall train students in the following skills for 15 hours, by taking relevant outside data (Field/Web).
- 1. Beta and Gamma functions, Chebyshev polynomials.
- 2. Power series, power series solutions of ordinary differential equations,
- 3. Procedures of finding series solutions of Hermite equation, Legendre equation and Bessel equation.
- 4. Procedures of finding generating functions for Hermite polynomials, Legendre Polynomials and Bessel's function.
- 2. For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work, make observations and conclusions and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the aspects.
- 1. Going through the web sources like Open Educational Resources on the properties of Beta and Gamma functions, Chebyshev polynomials, power series solutions of ordinary differential equations. (or)
- **2.** Going through the web sources like Open Educational Resources on the properties of series solutions of Hermite equation, Legendre equation and Bessel equation.
- 3. Max. Marks for Fieldwork/Project work Report: 05.
- 4. Suggested Format for Fieldwork/Project work Report: Title page, Student Details,

Index page,

Stepwise work-done, Findings, Conclusions and Acknowledgements.

5. Unit tests (IE).

b) Suggested Co-Curricular Activities:

- 1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
- 2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
- 3. Invited lectures and presentations on related topics by experts in the specified area.



B. A/B.Sc	Semester – V (Skill Enhancement Course- Elective)	Credits:
Course: 6B	Multiple Integrals And Applications Of Vector Calculus	Hrs/Wk:

Learning Outcomes:

Students after successful completion of the course will be able to

- 1. Learn multiple integrals as a natural extension of definite integral to a function of two variables in the case of double integral / three variables in the case of triple integral.
- 2. Learn applications in terms of finding surface area by double integral and volume by triple integral.
- 3. Determine the gradient, divergence and curl of a vector and vector identities.
- 4. Evaluate line, surface and volume integrals.
- 5. understand relation between surface and volume integrals (Gauss divergence theorem), relation between line integral and volume integral (Green's theorem), relation between line and surface integral (Stokes theorem)

Syllabus: (Hours: Teaching: 75 (incl. unit tests etc.05), Training: 15)

Unit – 1: Multiple integrals-I

(15h)

- 1. Introduction, Double integrals, Evaluation of double integrals, Properties of double integrals.
- 2. Region of integration, double integration in Polar Co-ordinates,
- 3. Change of variables in double integrals, change of order of integration.

Unit – 2: Multiple integrals-II

(15h)

- 1. Triple integral, region of integration, change of variables.
- 2. Plane areas by double integrals, surface area by double integral.
- 3. Volume as a double integral, volume as a triple integral.

Unit – 3: Vector differentiation

(15h)

- 1. Vector differentiation, ordinary derivatives of vectors.
- 2. Differentiability, Gradient, Divergence, Curl operators,
- 3. Formulae involving the separators.

Unit – 4: Vector integration

(15h)

- 1. Line Integrals with examples.
- 2. Surface Integral with examples.
- 3. Volume integral with examples.

Unit – 5: Vector integration applications

(15h)

- 1. Gauss theorem and applications of Gauss theorem.
- 2. Green's theorem in plane and applications of Green's theorem.
- 3. Stokes's theorem and applications of Stokes theorem.

Reference Books:

- 4.Dr.M Anitha, Linear Algebra and Vector Calculus for Engineer, Spectrum University Press, SR Nagar, Hyderabad-500038, INDIA.
- 5.Dr.M.Babu Prasad, Dr.K.Krishna Rao, D.Srinivasulu, Y.AdiNarayana, Engineering Mathematics-II, Spectrum University Press, SR Nagar, Hyderabad-500038,INDIA.
- 6. V.Venkateswararao, N. Krishnamurthy, B.V.S.S.Sarma and S.Anjaneya Sastry, A text Book of B.Sc., Mathematics Volume-III, S. Chand & Company, Pvt. Ltd., Ram Nagar,NewDelhi-110055.
- 7. R.Gupta, Vector Calculus, Laxmi Publications.
- 8. P.C.Matthews, Vector Calculus, Springer Verlag publications.
- 9. Web resources suggested by the teacher and college librarian including reading material.

Co-Curricular Activities:

A) Mandatory:

- 1. For Teacher: Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Field/Web).
 - 1. The methods of evaluating double integrals and triple integrals in the class room and train to evaluate

These integrals of different functions over different regions.

- **2.** Applications of line integral, surface integral and volume integral.
- **3.** Applications of Gauss divergence theorem, Green's theorem and Stokes's theorem.
- 2. For Student: Fieldwork/Project work Each student individually shall undertake Fieldwork/Project work and submit a

report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the following aspects.

- 1. Going through the web sources like Open Educational Resources to find the values of double and triple integrals of specific functions in a given region and make conclusions. (or)
- **2.** Going through the web sources like Open Educational Resources to evaluate line integral, surface integral and volume integral and apply Gauss divergence theorem, Green's theorem and Stokes theorem and make conclusions.
- 3. Max. Marks for Fieldwork/Project work Report: 05.
- **4.** Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

4. Unit tests (IE).

b) Suggested Co-Curricular Activities:

- 1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
- 2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
- 3. Invited lectures and presentations on related topics by experts in the specified are



B. A/B.Sc	Semester – V (Skill Enhancement Course- Elective)	Credits:
Course: 7B	Integral Transforms With Applications	Hrs/Wk:

Learning Outcomes:

Students after successful completion of the course will be able to

- 1. Evaluate Laplace transforms of certain functions, find Laplace transforms of derivatives and of integrals.
- 2. Determine properties of Laplace transform which may be solved by application of special functions namely Dirac delta function, error function, Bessel function and periodic function.
- 3. Understand properties of inverse Laplace transforms, find inverse Laplace transforms of derivatives and of integrals.
- 4. Solve ordinary differential equations with constant/ variable coefficients by using Laplace transform method.
- 5. Comprehend the properties of Fourier transforms and solve problems related to finite Fourier transforms.

Syllabus: (Hours: Teaching: 75 (incl. unit tests etc.05), Training: 15)

Unit – 1: Laplace transforms- I

(15h)

- 1. Definition of Laplace transform, linearity property-piecewise continuous function.
- 2. Existence of Laplace transform, functions of exponential order and of class A.
- **3.** First shifting theorem, second shifting theorem and change of scale property.

Unit – 2: Laplace transforms- II

(15h)

- 1. Laplace Transform of the derivatives, initial value theorem and final value theorem. Laplace transforms of integrals.
- 2. Laplace transform of t^n . f(t), division by t, evolution of integrals by Laplace transforms.
- 3. Laplace transform of some special functions-namely Dirac delta function, error function, Bessel function and Laplace transform of periodic function.

Unit – 3: Inverse Laplace transforms

(15h)

- 1. Definition of Inverse Laplace transform, linear property, first shifting theorem, second shifting theorem, change of scale property, use of partial fractions.
- 2. Inverse Laplace transforms of derivatives, inverse, Laplace transforms of integrals, multiplication by powers of 'p', division by 'p'.
- 3. Convolution, convolution theorem proof and applications.

Unit – 4: Applications of Laplace transforms

(15h)

- 1. Solutions of differential equations with constants coefficients, solutions of differential equations with variable coefficients.
- 2. Applications of Laplace transforms to integral equations- Abel's integral equation.
- 3. Converting the differential equations into integral equations, converting the integral equations into differential equations.

Unit – 5: Fourier transforms

(15h)

- 1. Integral transforms, Fourier integral theorem (without proof), Fourier sine and cosine integrals.
- 2. Properties of Fourier transforms, change of scale property, shifting property, modulation theorem. Convolution.
- 3. Convolution theorem for Fourier transform, Parseval's Identify, finite Fourier transforms.

Reference Books:

- 1. Dr. S.Sreenadh, S.Ranganatham, Dr.M.V.S.S.N.Prasad, Dr. V.Ramesh Babu, Fourier series and Integral Transforms, S. Chand & Company, Pvt. Ltd., Ram Nagar, New Delhi-110055.
- 2. A.R. Vasistha, Dr. R.K. Gupta, Laplace Transforms, Krishna Prakashan Media Pvt. Ltd.Meerut.
- 3. M.D.Raisinghania, H.C. Saxsena, H.K. Dass, Integral Transforms, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
- 4. Dr. J.K. Goyal, K.P. Gupta, Laplace and Fourier Transforms, Pragathi Prakashan, Meerut.
- 5. Shanthi Narayana, P.K. Mittal, A Course of Mathematical Analysis, S. Chand & Company Pvt. Ltd. Ram Nagar, New Delhi-110055.
- 6. Web resources suggested by the teacher and college librarian including reading material.

Co-Curricular Activities:

A) Mandatory:

- **1. For Teacher:** Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Field/Web).
 - 1. Demonstrate on sufficient conditions for the existence of the Laplace transform of a function.
 - 2. Evaluation of Laplace transforms and methods of finding Laplace transforms.
 - 3. Evaluations of Inverse Laplace transforms and methods of finding Inverse Laplace transforms.
 - 4. Fourier transforms and solutions of integral equations.
- 2. For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work and submit a report not exceeding 10 pages in the given format on the work-done in the areas like thefollowing, by choosing any one of the aspects.
 - 1. Going through the web sources like Open Educational Resources on Applications of Laplace transforms and Inverse Laplace transforms to find solutions of ordinary differential equations with constant /variable coefficients and make conclusions. (or)
 - 2. Going through the web sources like Open Educational Resources on Applications of convolution theorem to solve integral equations and make conclusions. (or)
 - 3. Going through the web source like Open Educational Resources on Applications of Fourier transforms to solve integral equations and make conclusions.
 - 4. Max. Marks for Fieldwork/Project work Report: 05.
- 3. Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index

page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

4. Unit tests (IE).

b) Suggested Co-Curricular Activities:

- 1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
- 2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
- 3. Invited lectures and presentations on related topics by experts in the specified area.



ADIKAVINANNAYAUNIVERSITY::RAJAHMAHENDRAVARAM

B. A/B.Sc Mathematics Syllabus (w.e.f:2020-21A.B)

B. A/B.Sc	Semester – V (Skill Enhancement Course- Elective)	Credits:
Course: 6C	Partial differential equations & Fourier series	Hrs/Wk:

Learning Outcomes:

Students after successful completion of the course will be able to

- 1. Classify partial differential equations, formation of partial differential equations and solve Cauchy's problem for first order equations.
- 2. Solve Lagrange's equations by various methods, find integral Surface passing through a given curve and Surfaces orthogonal to a given system of Surfaces.
- 3. Find solutions of nonlinear partial differential equations of order one by using Char pit's method.
- 4. Find solutions of nonlinear partial differential equations of order one by using Jacobi's method.
- 5. Understand Fourier series expansion of a function f(x) and Parseval's theorem.

Syllabus: (Hours: Teaching: 75 (incl. unit tests etc.05), Training: 15)

Unit – 1: Introduction of partial differential equations

(15h)

- 1. Partial Differential Equations, classification of first order partial differential equations, Rule I, derivation of a partial differential equations by the elimination of arbitrary constants
- 2. Rule II, derivation of a partial differential equation by the elimination of arbitrary function φ from the equations $\emptyset(u,v)=0$ where u and v are functions of x, y and z.
- 3. Cauchy's problem for first order equations

Unit – 2: Linear partial differential equations of order one

(15h)

- 1. Lagrange's equations, Lagrange's method of solving Pp+Qq=R, where P, Q and R are functions of x, y and z, type 1 based on Rule I for solving $\frac{dx}{p} = \frac{dy}{Q} = \frac{dz}{R}$, type 2 based on Rule II for solving $\frac{dx}{p} = \frac{dy}{Q} = \frac{dz}{R}$.
- 2. Type 3 based on Rule III for solving $\frac{dx}{p} = \frac{dy}{0} = \frac{dz}{R}$, type 4 based on Rule IV for $solving \frac{dx}{p} = \frac{dy}{0} = \frac{dz}{R}$
- 3. Integral Surface passing through a given curve, the Cauchy problem, Surfaces orthogonal to a given system of Surfaces.

Unit – 3: Non-linear partial differential equations of order one-I (15h)

- 1. Complete integral, particular integral, singular integral and general integral, geometrical interpretation of integrals of f(x, y, z, p, q) = 0, method of getting singular integral from the PDE of first order, compatible system of first order equations.
- 2. Char pit's method, Standard form I, only p and q present.
- 3. Standard form II, Clairaut equations.

Unit – 4: Non-linear partial differential equations of order one-II (15h)

- 1. Standard Form III, only p, q and z present.
- 2. Standard Form IV, equation of the form $f_1(x, p) = f_2(y, q)$.
- 3. Jacobi's method, Jacobi's method for solving partial differential equations with three or more independent variables, Jacobi's method for solving a non-linear first order partial differential equations in two independent variables.

Unit – 5: Fourier series (15h)

- 1. Introduction, Euler's formulae for Fourier series expansion of a function f(x), Dirichlet's conditions for Fourier series, convergence of Fourier series.
- 2. Functions having arbitrary periods. Change of interval, Half range series.
- 3. Parseval's theorem, illustrative examples based on Parseval's theorem, some particular series.

Reference Books:

- 1. Dr.M.D.Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
- 2. Dr. S.Sreenadh, S.Ranganatham, Dr.M.V.S.S.N.Prasad, Dr. V.Ramesh Babu, Fourier Series and Integral Transforms, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
- **3.** Prof T.Amaranath, An Elementary Course in Partial Differential Equations Second Edition, Narosa Publishing House, New Delhi.
- 4. Fritz John, Partial Differential Equations, Narosa Publishing House, New Delhi, 1979.
- **5.** I.N.Sneddon, Elements of Partial Differential Equations by McGraw Hill, International Edition, Mathematics series.
- 6. Web resources suggested by the teacher and college librarian including reading material.

Co-Curricular Activities:

A) Mandatory:

- **1. For Teacher:** Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Field/Web).
- 1. On classification of first order partial differential equations, formation of partial differential equations.
- 2. Various methods of finding solutions of partial differential equations.
- 3. Integral Surface passing through a given curve and Surfaces orthogonal to a give system of Surfaces.

b) For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work and submit a report not exceeding 10 pages in the given format on the workdone in the areas like the

Following, by choosing any one of the aspects.

- 1. Going through the web source like Open Educational Resources to find solutions of partial differential equations by using Lagrange's method, Charpit's method and Jacobi's method and make conclusions. (or)
- 2. Going through the web source like Open Educational Resources to find Integral Surface passing through a given curve and Surfaces orthogonal to a given system of Surfaces and make conclusions. (or)
- 3. Going through the web source like Open Educational Resources to find Fourier series expansions of some functions and applications of Parseval's theorem and make conclusions.
- 3. Max. Marks for Fieldwork/Project work Report: 05.
- **4.** Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.
 - 5. Unit tests (IE).
 - b) Suggested Co-Curricular Activities
 - 1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
 - 2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
 - 3. Invited lectures and presentations on related topics by experts in the specified area.



B. A/B.Sc	Semester – V (Skill Enhancement Course- Elective)	Credits:
Course: 7C	Number Theory	Hrs/Wk:

Learning Outcomes:

Students after successful completion of the course will be able to

- 4. Find quotients and remainders from integer division, study divisibility properties of integers and the distribution of primes.
- 5. Understand Dirichlet multiplication which helps to clarify interrelationship between various arithmetical functions.
- 6. Comprehend the behaviour of some arithmetical functions for large n.
- 7. Understand the concepts of congruencies, residue classes and complete residues systems.
- 8. Comprehend the concept of quadratic residues mod p and quadratic non residues mod p.

Syllabus: (Hours: Teaching:75 (incl. unit tests etc.05), Training:15)

Unit – 1: Divisibility (15h)

- 1. Introduction, Divisibility, Greatest Common Divisor.
- 2. Prime numbers, The fundamental theorem of arithmetic, The series of reciprocals of the primes.
- 3. The Euclidean algorithm, The greatest common divisor of more than two numbers.

Unit – 2: Arithmetical Functions and Dirichlet Multiplication (15h)

- 1. Introduction, The Mobius function $\mu(n)$, The Euler totient function $\phi(n)$, A relation connecting ϕ and μ , A product formula for $\phi(n)$.
- 2. The Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, The Mangoldt function $\Lambda(n)$.
- 3. Multiplicative functions, Multiplicative functions and Dirichlet multiplication, The inverse of a completely multiplicative function, Liouville's function $\lambda(n)$, The divisor functions $\sigma_{\alpha}(n)$.

Unit – 3: Averages of Arithmetical Functions

(15h)

- 1. Introduction, The big oh notation. Asymptotic equality of functions, Euler's summation formula, some elementary asymptotic formulas.
- 2. The average order of d(n), The average order of the divisor functions $\sigma_{\alpha}(n)$, The average order of $\varphi(n)$.
- 3. The average order of $\mu(n)$ and $\Lambda(n)$, The partial sum of a Dirichlet product, Applications of $\mu(n)$ and $\Lambda(n)$.

Unit – 4: Congruences

(15h)

- 1. Definition and basic properties of congruences, Residue classes and complete residue systems.
- 2. Linear congruences, reduced residue systems and the Euler-Fermat theorem. Polynomial congruences modulo p. Lagrange's theorem.
- 3. Applications of Lagrange's theorem, Simultaneous linear congruences. The Chinese remainder theorem. Applications of the Chinese remainder theorem.

Unit – 5: Quadratic Residues and the Quadratic Reciprocity Law (15h)

- 1. Quadratic Residues, Legendre's symbol and its properties, Evaluation of (-1/p) and (2/p), Gauss lemma,
- 2. The Quadratic reciprocity law, Applications of the reciprocity law, The Jacobi Symbol.
- **3.** Gauss sums and the quadratic reciprocity law, the reciprocity law for quadratic Gauss sums. Another proof of the quadratic reciprocity law.

Reference Books:

- 1. Tom M.Apostol, Introduction to Analytic Number theory, Springer International Student Edition.
- 2. David, M. Burton, Elementary Number Theory, 2nd Edition UBS Publishers.
- 3. Hardy & Wright, Number Theory, Oxford Univ, Press.
- 4. Dence, J. B & Dence T.P, Elements of the Theory of Numbers, Academic Press.
- 5. Niven, Zuckerman & Montgomery, Introduction to the Theory of Numbers.
- 6. Web resources suggested by the teacher and college librarian including reading material.

Co-Curricular Activities:

A) Mandatory:

- **1. For Teacher:** Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Field/Web).
 - 1. Finding quotient and numbers from integer division and the method of solving congruences. Further problems related to the theory of quadratic residues.
 - 2. Applications of Lagrange's theorem.
 - 3. Applications of the Chinese remainder theorem.
 - 4. Applications of the reciprocity law.
 - **2.For Student: Fieldwork/Project work;** Each student individually shall undertake Fieldwork/Project work and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the aspects.
 - 1. Going through the web sources like Open Educational Resources and list out Applications of Lagrange's theorem, and make conclusions.(or)
 - 2. Going through the web sources like Open Educational Resources and list out Applications of the Chinese remainder theorem and make conclusions.(or)
 - 3. Going through the web sources like Open Educational Resource and list out Applications of the reciprocity law and make conclusions.
 - 3. Max. Marks for Fieldwork/Project work Report: 05.
 - 4. Suggested Format for Fieldwork/Project work Report:

Title page, Student Details, Index page,

Step wise work-done, Findings, Conclusions and Acknowledgements.

5. Unit tests (IE).

b) Suggested Co-Curricular Activities

- 1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
- 2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
- 3. Invited lectures and presentations on related topics by experts in the specified area.

ADIKAVI NANNAYYA UNIVERSITY :: RAJAMAHENDRAVARAM CBCS/ SEMESTER SYSTEM

(W. e. f 2020 – 21 Admitted Batch) B. A./B. Sc. MATHEMATICS

$COURSE-VI(A), NUMERICAL\ METHODS.$

MATHEMATICS MODEL PAPER

Max. Marks: 75M Time: 3Hrs

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks.

5 X 5 M = 25 M

- 1) Find the function whose first difference is $9x^2 + 11x + 5$.
- 2) Find the missing term in the following table

X	0	1	2	3	4
У	1	1.5	2.2	3.1	4.6

- 3) If $f(x) = \frac{1}{x^2}$ then find the divided differences f(a,b) and f(a,b,c).
- 4) Using Gauss forward interpolation formula to find f(2.5) from the following table.

X	1	2	3	4
f(x)	1	8	27	64

- 5) Derive the derivative $\left(\frac{dy}{dx}\right)_{x=x_0}$ by using Newton's backward interpolation formula.
- 6) Find $\frac{dy}{dx}$ at x = 0, using the table

Х	0	2	4	6	8	10
f(x)	0	12	248	1284	4080	9980

- 7) Evaluate the integral $\int_0^6 \frac{dx}{1+x}$ by using Simpson's $\frac{1}{3}$ rule.
- 8) Using Taylor's series method, solve the equation $\frac{dy}{dx}=x^2+y^2$ for x=0.4, given that y=0 when x=0.

SECTION - B

Answer any ALL questions. Each question carries TEN marks.

5 X 10 M = 50 M

9 a) State and Prove Newton's forward interpolation formula.

OF

9 b) Show that i)
$$\mu^2 = 1 + \frac{1}{4}\delta^2$$
 and ii) $1 + \mu^2 \delta^2 = \left(1 + \frac{1}{2}\delta^2\right)^2$

OR

10 b) Using Lagrange's formula fit a polynomial to the following data and hence find f(1).

X	-1	0	2	3
f(x)	8	3	1	12

11 a) Derive the derivatives $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=x_0$ by using Stirling's interpolation formula.

OR

11 b) Compute $f^1(4)$ and $f^1(5)$ from the following table

X	1	2	4	8	10
f(x)	0	1	5	21	27

12 a) State and prove General Quadrature Formula.

OR

- 12 b) Evaluate the integral $\int_0^6 \frac{dx}{1+x^3} dx$ by using Weddle's rule.
- 13 a) Use Runge Kutta method to evaluate y (0.1) and y (0.2) given that $y^1 = x + y$, initial condition y(0) = 1.

OR

13 b) Given $\frac{dy}{dx} = x + y$ with initial condition y(0) = 1. Find y(0.05) and y(0.1), correct to 6 decimal places by using Euler's modified method.

ADIKAVI NANNAYA UNIVERSITY, RAJAMAHENDRAVARAM

B.A./B.Sc., FIFTH SEMESTER MATHEMATICS MODEL PAPER 7A: MATHEMATICAL SPECIAL FUNCTIONS

(w. e. f. 2020-21 admitted batch)

TIME: 3hrs

MAX.MARKS:75

SECTION- A

Answer any FIVE questions. Each question carries 5 marks.

5 X 5 = 25 Marks

- 1. Evaluate $\int_0^2 \frac{x^2 dx}{\sqrt{(2-x)}}$
- 2. Show that $\Gamma\left(\frac{1}{2} + x\right) \Gamma\left(\frac{1}{2} x\right) = \frac{\pi}{\cos \pi x}$
- 3. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{R}$ then prove that $\sum a_n x^n$ is convergent for |x| < R and divergent for |x| > R
- 4. Prove that $H''_n(x) = 4n(n-1) H_{n-1}(x)$
- 5. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$
- 6. Prove that $P_n(-x) = (-1)^n P_n(x)$
- 7. Prove that $P'_n(1) = \frac{1}{2}n(n+1)$
- 8. Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is a positive integer

SECTION -B

Answer any FIVE questions. Each question carries 10 marks.

5 X 10 = 50 Marks

9(a). Prove that
$$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

OR

- 9(b). Prove that $2^n \Gamma\left(n + \frac{1}{2}\right) = 1.3.5 \dots (2n-1)\sqrt{\pi}$ where *n* is a positive integer
- 10(a). Solve y' y = 0 by power series method

OR

10(b). Find the power series solution in powers of (x-1) of the initial value problem

$$xy'' + y' + 2y = 0, y(1) = 1, y'(1) = 2.$$

11(a). Prove that
$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} (e^{-x^2})$$

OR

11(b). Prove that
$$2xH_n(x) = 2n H_{n-1}(x) + H_{n+1}(x)$$

12(a). Prove that
$$(1 - 2xh + h^2)^{-1/2} = \sum_{n=0}^{\infty} h^n P_n(x)$$

OR

12(b).
$$\int_{-1}^{1} P_m(x) P_n(x) dx = 0 \text{ if } m \neq n$$

13(a).
$$xJ'_n(x) = n J_n(x) - x J_{n+1}(x)$$

OR

13(b). Show that
$$J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$$

MODEL QUESTION PAPER (Sem-End) B.A./B.Sc. DEGREE EXAMINATIONS

Semester –V (Skill Enhancement Course-Elective)

Course 6B: MULTIPLE INTEGRALS & APPLICATION OF VECTOR CALICULUS

Time: 3Hrs Max.Marks:75M

SECTION - A

Answer any FIVE questions.

5 X 5M=25 M

- 1. Evaluate $\int_0^a \int_0^b xy(x^2 + y^2) dx dy$
- 2. Evaluate $\int_0^1 \int_{-1}^0 \int_{-1}^1 (x+y+1) dx dy dz$
- 3. If $\bar{F} = (x + 3y)\hat{i} + 9y 2z)\hat{j} + (x + pz)\hat{k}$ is a solenoidal find p.
- 4. Prove that $\Delta \times (\Delta \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) \nabla^2 \bar{A}$
- 5. Find the value of $\int (x+y^2)dx + (x-y^2)dy$, taken in the clockwise direction along the closed curve C formed by $y^2 = x$ and y = x between (0,0) and (0,1).
- 6. Evaluate $\int \overline{F} dr$, where $\overline{F} = x^2 y^2 \mathbf{i} + y \mathbf{j}$ and the curve C is $y^2 = 4x$ in the XY-plane from (0,0) to (4,4)
- 7. Show that $\int_{S} (ax\mathbf{i} + by\mathbf{j} + cz\mathbf{k}). \, \bar{n}ds = \frac{4\pi}{3}(a+b+c)$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$
- 8. By Stokes theorem evaluate $\int_C ydx + zdy + xdz$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and x + z = a

SECTION - B

Answer ALL the questions.

5 X 10 M = 50 M

- 9. a) Evaluate $\iint_S xydxdy$, where S is the region bounded by xy = 1, y = 0, y = x, x = 2 (OR)
 - b) Change the order of integration and hence show that

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dy}{(1+e^y)\sqrt{1-x^2-y^2}} = \frac{\pi}{2} \log \left[\frac{2e}{1+e} \right]$$

10. a) Using double integration find the valume of solid bounded by the coordinate plane x = 0, y = 0, z = 0 and plane x + y + z = 1 (OR)

b) Evaluate $\iiint xyzdxdydz$ taken over the cube bounded by the planes x=0, y=0, z=0 and x=2, y=2, z=2 in the first octant.

11. a) If \bar{a} is a constant vector, prove that $\operatorname{curl} \frac{\bar{a} \times \bar{r}}{r^3} = -\frac{\bar{a}}{r^3} + \frac{3\bar{r}}{r^5} (\bar{a}.\bar{r})$ (OR)

B.A/B.Sc Mathematics Page 26 of 26

- b) Prove that $grad(\bar{A}\bar{B}) = (\bar{B}.\nabla)\bar{A} + (\bar{A}.\nabla)\bar{B} + \bar{B} \times curl\bar{A} + \bar{A} \times curl\bar{B}$
- 12. a) Evaluate $\int_S \bar{F} \bar{N} ds$, where $\bar{F} = z \mathbf{i} + x \mathbf{j} 3y^2 z \mathbf{k}$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between z = 0 and z = 5 (OR)
 - b) If $\bar{F} = 2xz\mathbf{i} x\mathbf{j} + y^2\mathbf{k}$, evaluate $\int_V \bar{F} dv$ where V is the region bounded by the surface $x = 0, x = 2, y = 0, y = 6, z = x^2, z = 4$
- 13. a) State and prove *Gauss's divergence theorem* (OR)
 - b) Verify Stokes theorem for $\overline{F} = -y^3 \mathbf{i} + x^3 \mathbf{j}$, where S is the circular disc $x^2 + y^2 \le 1, z = 0$

B.A/B.Sc Mathematics Page 26 of 26

ADIKAVI NANNAYYA UNIVERSITY :: RAJAMAHENDRAVARAM **CBCS/ SEMESTER SYSTEM**

(W. e. f 2020 - 21 Admitted Batch)B. A./B. Sc. MATHEMATICS

COURSE – VII(B), INTEGRAL TRANSFORMS WITH APPLICATIONS

MATHEMATICS MODEL PAPER

Max. Marks: 75M Time: 3Hrs

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks.

5 X 5 M = 25 M

1. Find L[f(t)], where f(t) =
$$\begin{cases} \sin\left(t - \frac{2\pi}{3}\right) & \text{if } t > \frac{2\pi}{3} \\ 1 & \text{if } t < \frac{2\pi}{3} \end{cases}$$

- 2.. State and prove second shifting property of Laplace Transform.
- 3.. State Bessel's function and hence show that $L[J_0(a\sqrt{t})] = \frac{e^{-(\frac{a^2}{s})}}{c}$
- 4. Find $L^{-1} \left[\frac{p^2}{(p-3)^2} \right]$ 5. Evaluate $\int_{0}^{\infty} \frac{\sin 2t}{t} dt$
- 6. Find $L^{-1}\left[\frac{3p+1}{(p-1)(p^2+1)}\right]$ by using partial fractions.
- 7. Solve $(D^2 D 2)y = 20 \sin 2t$ if y = 1, Dy = 2 when t = 0 by the method of Laplace Transform.
- 8. Find the sine and cosine transform of the function f(x) = x.

SECTION - B

Answer any ALL questions. Each question carries TEN marks.

5 X 10 M = 50 M

9. a) Evaluate $L[t^2 e^{-2t} \cos t]$

OR

9. b) State and prove first shifting theorem and also find L[sin hat cos at]

10 . a) Show that
$$L\left(\frac{sint}{t}\right) = tan^{-1}\left(\frac{1}{s}\right)$$
 and from this find the value of $L\left(\frac{sin\ at}{t}\right)$. Does $L\left(\frac{cos\ at}{t}\right)$ exists ?

OR

10. b) Show that
$$\int_{0}^{\infty} t^{3} e^{-t} \sin t dt = 0$$

11. a) State and prove Convolution theorem

OR

11. b) Evaluate
$$L^{-1}\left[\frac{2s+t}{(s+2)^2(s^2-1)}\right]$$

12. a) Solve $(D^3 + 2D^2 - D - 1)y = 0$ given $y(0) = y^1(0) = 0$ and $y^{11}(0) = 6$ by the method of Laplace Transform.

12. b) Solve the integral equation
$$f(t) = t + 2 \int_{0}^{t} \cos(t - u)F(u)du = 0$$

13. a) Find the finite cosine transform of
$$\left(1 - \frac{x}{\pi}\right)^2$$
 where $0 < x < \pi$. OR

13. b) Find the fourier sine function of $\frac{e^{-ax}}{x}$ and hence deduce that

$$\int_{0}^{\infty} \left(\frac{e^{-ax} - e^{-bx}}{x} \right) \operatorname{sinpx} dx = \tan^{-1} \left(\frac{p}{a} \right) - \tan^{-1} \left(\frac{p}{b} \right).$$

MODEL QUESTION PAPER (Sem-End) B.A./B.Sc. DEGREE EXAMINATIONS

Semester –V (Skill Enhancement Course-Elective)

Course 6C: Partial Differential Equations and Fourier series

Time: 3Hrs Max.Marks:75M

SECTION - A

Answer any FIVE questions.

1. Solve $z = f(x^2 + y^2)$

- 2. Solve (-a + x)p + (-b + y)q = (-c + z)
- 3. Solve $-qy \log y = z \log y$
- 4. Solve using Charpit's method pq = xz
- 5. Find the complete integral of the partial differential equation $p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$
- 6. If $z = e^{(ax+by)} f(ax by)$, then show that $a\left(\frac{\partial z}{\partial y}\right) + b\left(\frac{\partial z}{\partial x}\right) = 2abz$
- 7. Solve zp = -x
- 8. Dirichlet's conditions for Fourier series

SECTION - B

Answer ALL the questions.

5 X 10 M = 50 M

 $5 \times 5M = 25 M$

- 9. a) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$
 - b) Solve the Cauchy's problem for zp + q = 1, where the initial data curve is $x_0 = \mu$, $y_0 = \mu$, $z_0 = \frac{\mu}{2}$ $0 \le \mu \le 1$
- 10. a) Solve $x(x^2 + 3y^2)p y(3x^2 + y^2)q = 2z(y^2 x^2)$
 - b) Find the equation of the integral surface of the differential equation 2y(z-3)p +(2x-z)q = y(2x-3), which pass through the circle z = 0, $x^2 + y^2 = 2x$
- 11. a) Show that the equation z = px + qy is compatible with any equation f(x, y, z, p, q) = 0 which is homogeneous equation in x, y, z
 - b) Find the complete integral of $(x^2 + y^2)(p^2 + q^2) = 1$
- 12. a) Find complete integral of $2p_1x_1x_3 + 3p_2x_3^2 + p_2^2p_3 = 0$ (OR)
 - b) Solve $p^2 + q^2 = k^2$, by Jacobi's method
- 13. a) Find the Fourier series for f(x) define by f(x) = x for 0 < x < 1 and f(x) = 1 - x for 1 < x < 2. Deduce $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$
- b) Apply Parsvel's identity to the function f(x) = x, $-\pi \le x \le \pi$ and deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} + \dots$ B.A/B.Sc Mathematics

Page 26 of 26

ADIKAVI NANNAYA UNIVERSITY, RAJAMAHENDRAVARAM

B.A./B.Sc., FIFTH SEMESTER MATHEMATICS MODEL PAPER

7C: NUMBER THEORY

(w. e. f. 2020-21 admitted batch)

TIME: 3hrs MAX.MARKS:75

SECTION- A

Answer any **FIVE** questions. Each question carries 5 marks.

5 X 5 = 25 Marks

- 1. If a prime p divides ab then p/a or p/b
- 2. If $n \ge 1$ then $\log n = \sum_{d/n} \wedge (d)$
- 3. If both g and f*g are multiplicative then f is also multiplicative
- 4. Show that for $x \ge 1$, $\sum_{n \le x} \mu(n) \left[\frac{x}{n} \right] = 1$
- 5. Show that for $x \ge 2$, $\sum_{p \le x} \left[\frac{x}{p} \right] \log p = x \log x + O(x)$ where the sum is extended over all primes $\le x$.
- 6. For any integer a and any prime p then Prove that $a^p \equiv a \pmod{p}$
- 7. If (a, m) = 1 then prove that the linear congruence $ax \equiv b \pmod{m}$ has exactly one solution.
- 8. For every odd prime p, $(s/p) = (-1)^{p^2-1/8} = \begin{cases} 1 & \text{if } p \equiv \pm 1 \pmod{8} \\ -1 & \text{if } p \equiv \pm 3 \pmod{8} \end{cases}$

SECTION-B

Answer any **FIVE** questions. Each question carries 10 marks.

5 X 10 = 50 Marks

9(a). State and prove fundamental theorem of arithmetic.

OR

9(b). State and prove the division algorithm.

10(a). If
$$n \ge 1$$
 then $\phi(n) = \sum_{d/n} \mu(d) \frac{n}{d}$

OR

10(b). State and prove Mobius inversion formula

11(a). State and prove Eulers summation formula

OR

11(b). For
$$x > 1$$
, $\sum_{n \le x} \Phi(n) = \frac{3}{\Pi^2} x^2 + O(x \log x)$

12(a). State and prove Lagrange's theorem

OR

- 12(b). State and prove Chinese remainder theorem
- 13(a). State and prove Gauss lemma

OR

13(b). State and prove Quadratic Reciprocal Law
